# Ground state of the Universe and the cosmological constant. A nonperturbative analysis

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The physical Hamiltonian of a gravity-matter system depends on the choice of time, with the vacuum naturally identified as its ground state. We study the expanding universe with scalar field in the volume time gauge. We show that the vacuum energy density computed from the resulting Hamiltonian is a non-linear function of the cosmological constant and time. This result provides a new perspective on the relation between time, the cosmological constant, and vacuum energy.

One of the outstanding problems in fundamental physics is that of the cosmological constant (CC). The problem arises in the context of quantum field theory (QFT) on a fixed background spacetime, which is usually taken to be flat [1–4], or otherwise has a high degree of symmetry. The symmetry includes a global notion of time specified as a timelike Killing vector field. The dynamics of the gravitational field is included only in so far as it is viewed as a spin two field on the specified background; back reaction of quantum fields on spacetime is typically excluded.

QFT on a fixed background spacetime may be viewed as the leading order term coming from the semi-classical approximation defined by the equation

$$G_{ab} + \Lambda g_{ab} = 8\pi G \langle\psi|\hat{T}_{ab}(\hat{\phi}, g)|\psi\rangle, \tag{1}$$

where $\Lambda$ is the (bare) cosmological constant. As written, this hybrid classical-quantum equation is ambiguous. To make it more precise we require (i) a quantization of the matter field $\phi$ on a general background $g_{ab}$, (ii) a suitably regularized self-adjoint operator $\hat{T}_{ab}$, and lastly (iii) computation of the expectation value of $\hat{T}_{ab}$ in some choice of matter vacuum state $|\psi\rangle$. This would give the tensor

$$\tilde{T}^{\psi}_{ab}(g) \equiv \langle\psi|\hat{T}_{ab}(\hat{\phi}, g)|\psi\rangle, \tag{2}$$

as the effective stress-energy tensor associated to the state $|\psi\rangle$, and hence a precise meaning for the r.h.s. of eqn. (1). One can then proceed to solve this equation for the "semi-classical" metric $g_{ab}$.

Although there is a large literature [5] on computations of the r.h.s for a given spacetime, the calculation of a semiclassical metric has not been carried to satisfactory completion, even for spacetimes with isometries. In fact the equation itself has been questioned [6]. Nevertheless, an attempt to produce a self-consistent solution by expanding the metric and state as

$$\begin{aligned} g_{ab} &= \eta_{ab} + \epsilon h^{(1)}_{ab} + \epsilon^2 h^{(2)} + \cdots \\ |\psi\rangle &= |0\rangle + \epsilon|\psi^{(1)}\rangle + \epsilon^2|\psi^{(2)}\rangle + \cdots \end{aligned} \tag{3}$$

($\epsilon = m/m_P$) leads to $0th.$ order to

$$\Lambda\eta_{ab} = 8\pi G\langle 0|T_{ab}|0\rangle. \tag{4}$$

This equation forms the basis of the connection between vacuum energy density $\rho_{vac}$ and $\Lambda$, specifically the broadly accepted linear relationship

$$\rho_{vac} = \frac{\Lambda}{8\pi G}. \tag{5}$$

It leads to the cosmological constant problem via the elementary evaluation

$$\rho_{vac} = \frac{E}{V} = \int_0^{k_p} \frac{d^3k}{(2\pi)^3}(\hbar k) = \frac{\hbar}{8\pi^2}k_p^4, \tag{6}$$

where $k_p$ is a Planck scale cutoff. This huge quantity is often compared to the observed WMAP value

$$\Lambda = 1.27 \pm 0.07 \times 10^{-56}\ \mathrm{cm}^{-2} \tag{7}$$

($\sim 3.2 \times 10^{-122}\ l_P^{-2}$) as a significant failure of theory.

A more sophisticated argument presents this issue as a problem coming from running scales in the theory. Assuming a fixed background that defines energy $k$, the regulated vacuum energy density computed from $\langle 0|\hat{T}_{ab}|0\rangle$ is expected (on dimensional grounds) to be of the form

$$\rho_{vac} = M^4 f(k; g_1, g_2, \cdots) = \frac{\Lambda(k)}{8\pi G(k)}. \tag{8}$$

where $f$ is a function of energy scale $k$, matter coupling constants $g_1, g_2 \cdots$, and some natural mass scale $M(k)$. The first equality comes from field theory, and the second from semiclassical general relativity. (This expression assumes the usual linear dependence of energy density on $\Lambda$, and is observer 4-velocity $v^a$ dependent: $\rho = v^a v^b\langle 0|\hat{T}_{ab}|0\rangle$, unless there is a preferred timelike vector field specified by a spacetime isometry.) In

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this setting there are two ways to state the CC problem: (i) it arises from the first equality due to the factor $M^4$ which gives a very large energy density even well below the Planck energy, for example for proton mass or $\Lambda_{QCD}$, or (ii) it arises from the second equality as a fine tuning problem; at low energies (1 meter to a few astronomical units) where $G$ and $\Lambda$ are observed to be constant, the corresponding dimensionless parameters flow canonically as $\lambda(k) = \Lambda/k^2$ and $g(k) = Gk^2$. Thus the low energy renormalization group trajectory must be a hyperbola $\lambda(k)g(k) =$ constant, which reflects a fine tuning of the initial conditions for the flow [7].

The field theory problem may be due to the fact that the function $f$ is usually computed in perturbation theory. A counterpoint is provided by a recent non-perturbative calculation in the Gross-Neveu model, which suggests that, non-perturbatively, $f$ is a non-analytic function of the coupling constant that suppresses $\rho_{vac}$ at low energy [8].

We question the basis of formulating the CC problem to first order in the semiclassical setting, and argue that in a non-perturbative quantum approach in which gravitational degrees of freedom are treated as a part of the dynamics, either the problem does not arise, or that its manifestation is substantially different from that coming from the usual arguments.

We take the view that to meaningfully talk about a vacuum, we need a physical Hamiltonian for the full gravity-matter system. This in turn requires a global notion of time in the context of a generally covariant theory. Hence *there is a connection between non-perturbative vacuum energy, the cosmological constant, and a global time variable.* However as there is no "solution to the problem of time" in quantum gravity, one might impose a plausible time gauge, or use some other suitably defined "relational time." We will use geometry degrees of freedom to fix time gauge and derive the corresponding physical Hamiltonian. The spectrum of the corresponding operator then gives a formula for the vacuum energy density.

The suggestion that quantum gravity might play a role in its resolution is not new; see eg. [9] in the context of string theory, [10] in the Hamiltonian context which is developed further here, and a semiclassical approach using Regge calculus [11].

With this summary and context, we begin with the 3+1 Arnowitt-Deser-Misner (ADM) Hamiltonian for Einstein gravity and minimally coupled to a massive scalar field

$$S = \int d^3x dt \left(\pi^{ab}\dot{q}_{ab} + P_\phi\dot{\phi} - NH - N^a C_a\right), \qquad (9)$$

where $(q_{ab}, \pi^{ab})$ and $(\phi, P_\phi)$ are the ADM gravitational and scalar field phase space variables, $N, N^a$ are the lapses and shift variables, and

$$H = \frac{1}{\sqrt{q}}\left(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2\right) + \sqrt{q}(\Lambda - R) + H_\phi \qquad (10)$$

$$C_a = D_b \pi^b{}_a + P_\phi \partial_a \phi, \qquad (11)$$

$$H_\phi = \frac{1}{2}\left(\frac{P_\phi^2}{\sqrt{q}} + \sqrt{q}q^{ab}\partial_a\phi\partial_b\phi + \sqrt{q}m^2\phi^2\right). \qquad (12)$$

are respectively the Hamiltonian and diffeomorphism constraints, and the scalar field Hamiltonian density. (We work in geometric units where $G = \hbar = c = 1$, and reintroduce these constants in the final result.)

From this starting point, our goal is to calculate the vacuum energy density of the scalar field $\rho_{vac}(\Lambda, m)$ derived from the physical Hamiltonian associated to the volume time gauge in a cosmological setting. We do this first in the homogeneous (zero mode) setting to illustrate the argument, and subsequently generalize it to include all matter modes.

The flat homogeneous model is derived by the parametrization

$$q_{ab} = a^2 e_{ab}, \qquad \pi^{ab} = \frac{p_a}{6a}\, e^{ab} \qquad (13)$$

where $e_{ab} = \mathrm{diag}(1,1,1)$. Substituting this into the constraints and ADM action gives the reduced theory

$$S = V_0 \int dt \left(\dot{a}p_a + \dot{\phi}P_\phi - NH\right), \qquad (14)$$

where

$$H = -\frac{p_a^2}{24a} + a^3\Lambda + \frac{1}{2}\left(\frac{P_\phi^2}{a^3} + a^3 m^2\phi^2\right). \qquad (15)$$

The last equation is obtained from substituting the reduction ansatz into the Hamiltonain constraint (10), and $V_0$ is an unphysical coordinate volume. The reduced action is invariant under the scale transformations

$$(V_0, a, p_a, \phi, P_\phi) \rightarrow \left(\lambda^3 V_0, \frac{a}{\lambda}, \frac{p_a}{\lambda^2}, \phi, \frac{P_\phi}{\lambda^3}\right) \qquad (16)$$

At this stage we fix "physical volume time" gauge [12] by setting

$$t = \int d^3x\sqrt{q} = V_0 a^3. \qquad (17)$$

We note that this is both scale invariant (16) and second class with the Hamiltonian constraint, as required of an adequate gauge fixing. It is also a physically natural time in the context of an expanding cosmology. Although we do not require it here, the lapse function corresponding to this canonical time gauge is given by the requirement that the gauge be preserved under evolution. This gives

$$1 = \{V_0 a^3, NH\} = -\frac{V_0 N a p_a}{4} \implies N = -\frac{4}{V_0 a p_a}. \qquad (18)$$

We note that this lapse is invariant under the transformation (16), as it should be.

This gauge condition, together with the solution of the Hamiltonian constraint, eliminate the variables $(a, p_a)$, leaving a theory for the scalar field variables evolving with respect to this time. The gauge fixed canonical action is obtained by substituting (17) and the solution of the Hamiltonian constraint

$$p_a^2 = 24\left[a^4\Lambda + \frac{a}{2}\left(\frac{P_\phi^2}{a^3} + a^3 m^2\phi^2\right)\right]_{|V_0a^3=t} \quad (19)$$

into the action (14). We choose the root that gives positive energy density.

It is useful to write the gauge fixed action using the scale invariant variables $p_\phi := V_0 P_\phi$ and $t$. This gives

$$S^{GF} = \int dt\ \left(\dot{\phi}p_\phi - H_P\right), \quad (20)$$

where

$$H_P = \sqrt{\frac{8}{3}\left(\Lambda + \frac{p_\phi^2}{2t^2} + \frac{1}{2}m^2\phi^2\right)} \quad (21)$$

The energy density derived from this Hamiltonian is

$$\rho = \frac{H_P}{V_0 a^3} = \frac{H_P}{t}, \quad (22)$$

since $V_0 a^3$ is the physical volume (which is also the chosen time gauge). We note that this physical quantity does not depend on $V_0$.

To find the eigenvalues of this density operator we recall that for any operator $\hat{A}$ with a positive spectrum $a_n$, the spectrum of the square root operator $\sqrt{\hat{A}}$, is $\sqrt{a_n}$. In our case the argument of the square root in (21) is a shifted harmonic oscillator with time dependent mass and frequency. Therefore we can solve the eigenvalue problem for the density operator $\hat{\rho}$

$$\hat{\rho}\ \psi = \rho_n \psi \quad (23)$$

by treating $t$ as a parameter. This gives the exact spectrum

$$\rho_n = \frac{m_p^2}{t}\sqrt{\frac{8}{3}\left[\Lambda + \left(n + \frac{1}{2}\right)\frac{m}{t}\right]}, \quad (24)$$

where $n = 0, 1, \cdots$, and we have reintroduced the Planck mass, with $\Lambda, m$, and $t$ specified in Planck units.

Let us note that this energy density operator may also be used to set up the time dependent Schrodinger equation, specify an initial state, such as the $n = 0$ state, and evolve it to the present time. In general, such an evolved state may be approximated by a finite linear combination of the instantaneous energy eigenbasis $|\psi_n\rangle$ of the density operator,

$$|\Psi(t)\rangle = \sum_{n=0}^{N} c_n(t)|\psi_n(t)\rangle. \quad (25)$$

Now for our purpose, which is to obtain a relationship between energy density and cosmological constant, we would need to evaluate the expectation value of the density operator in this state

$$\langle\Psi(t)|\hat{\rho}(t)|\Psi(t)\rangle = \sum_{n=0}^{N} c_n(t)\rho_n(t). \quad (26)$$

However this is not necessary to make the central point of the paper, as we now show.

The expression gives a non-perturbative quantum energy density of the scalar field with respect to the volume time gauge (17). The eigenvalue $\rho_n(t)$ in this formula has some interesting features: (i) it depends only on variables invariant under the scale transformations (16), (ii) there is a square root arising from the fact that all terms in the Hamiltonian constraint are quadratic in momenta, (iii) the energy density is not linear in $\Lambda$, (iv) there is a time factor suppression which for large times gives

$$\rho_{vac} \equiv m_P^2\ \sqrt{\frac{8\Lambda}{3t^2}}, \quad (27)$$

independent of $n$.

These features are not what are expected from the usual flat space arguments for matter vacuum energy density, where this density is linear in $\Lambda$ and time independent. The last formula may be viewed as a prediction for the (zero mode) quantum vacuum energy density of the scalar field in an FRW universe, since this factor comes out of the sum (26) for late times. (We note that at each $t$ the state lives in the instantaneous Hilbert space at that time, so the remaining sum adds to unity. )

A numerical estimate of $\rho_{vac}$ using known cosmological parameters may be computed using the measured WMAP value for $\Lambda$ in eqn. (7) and the present age of the universe $t = 10^{61} t_P$, ($t_P$ = Planck time). This gives

$$\rho_{vac} \sim 5 \times 10^{-129}\rho_P = 2.5 \times 10^{-32}\ \mathrm{Kg/m^3}, \quad (28)$$

where $\rho_P = m_P/l_P^3$ is the Planck density. (We note that experiments such as WMAP measure cosmological model parameters such as $\Lambda$; implications for vacuum energy density are then derived from theoretical models. That is, there is no direct measurement of the energy density in a box of empty space.)

In summary to this point, we have seen that the time dependence in (24) has its origin in factors of $\int d^3x\sqrt{q} = V_0 a^3 = t$; the overall factor $1/t$ comes from converting the Hamiltonian scalar density (of weight one) to a scalar, and the factor in the oscillator frequency comes from the $\sqrt{q}$ terms in the matter Hamiltonian.

The semiclassical calculation of energy density is via $\rho = \langle 0|T_{ab} - \Lambda g_{ab}|0\rangle v^a v^b \equiv \rho_\phi + \rho_\Lambda$ for an observer with four velocity $v^a$. How is this to be compared with our result eqn. (24)? It is clear that the latter is additive in the contributions from matter and $\Lambda$, whereas our result (24) is not. It shows that imposing a time gauge, solving the Hamiltonian constraint, and then diagonalizing

the resulting physical Hamiltonian is an entirely different process from QFT on a fixed background, and yields substantially different results.

The setting we have discussed so far is obviously limited without a field theory extension to include all matter modes. This requires inclusion of inhomogeneities in the matter and metric degrees of freedom. We now turn to this. We will see that the main features of the energy density formula (24) – explicit time dependence and the square root – remain unaltered.

We follow a hamiltonian approach similar to that developed in [13], where the scalar field and metric perturbations are expanded in Fourier modes, and the Hamiltonian constraint is treated to second order in the perturbations. The resulting theory describes the dynamics of the gravity phase space variables $(a, p_a)$, and the scalar field and metric perturbation Fourier mode pairs $(\phi_k, p_k)$ and $(\delta q^k_{ab}, \delta\pi^{ab}_k)$; the mode decomposition is defined using the global chart on homogeneous space slices:

$$\phi(\mathbf{x}, t) = \sum_{\mathbf{k}} \phi_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$
$$P_\phi(\mathbf{x}, t) = \sum_{\mathbf{k}} P_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{29}$$

This gives

$$H_\phi = V_0 \sum_{\mathbf{k}} \left( \frac{P^2_{\mathbf{k}}}{2a^3} + \frac{a|\mathbf{k}|^2}{2}\phi^2_{\mathbf{k}} + \frac{a^3 m^2}{2}\phi^2_{\mathbf{k}} \right) \tag{30}$$

after a suitable mode relabelling. The Fourier modes so defined satisfy the equal time Poisson bracket

$$\{\phi_{\mathbf{k}}(t), P_{\mathbf{k}'}(t)\} = \delta_{\mathbf{k},\mathbf{k}'}. \tag{31}$$

With this decomposition we define a Hamiltonian system by the phase space variables $(a, p_a)$ and $(\phi_{\mathbf{k}}, P_{\mathbf{k}})$ and action

$$S = V_0 \int dt \left( \dot{a} p_a + \sum_{\mathbf{k}} \dot{\phi}_{\mathbf{k}} P_{\mathbf{k}} - NH \right). \tag{32}$$

where

$$H \equiv -\frac{p_a^2}{24a} + a^3\Lambda + \bar{H}_\phi = 0, \tag{33}$$

with $\bar{H}_\phi = H_\phi/V_0$ from (30). This Hamiltonian constraint generalizes (15) to include an infinite number of degrees of freedom.

This system is not exactly that obtained from metric and matter perturbations in [13]; in particular it does not include the spatial diffeomorphism constraint, which would impose further conditions between the matter and gravity modes. Nevertheless it is a consistent model that has the main features of interest for our purpose, which is to investigate the vacuum energy density of a matter-gravity system with an infinite number of degrees of freedom. We note also that the action (32) has the scaling invariance (16) with $(\phi, P_\phi)$ replaced by their Fourier modes $(\phi_{\mathbf{k}}, P_{\mathbf{k}})$.

Proceeding as for the homogeneous case, let us fix the (scale invariant) time gauge (17) and solve the Hamiltonian constraint to eliminate $(a, p_a)$. Using the scale invariant momentum $p_{\mathbf{k}} \equiv V_0 P_{\mathbf{k}}$, the gauge fixed action for the matter modes is

$$S^{GF} = \int dt \sum_k \left( \dot{\phi}_{\mathbf{k}} p_{\mathbf{k}} - H_P \right), \tag{34}$$

$$H_P = \sqrt{\frac{8}{3}\left( \Lambda + \sum_{\mathbf{k}} \left[ \frac{p^2_{\mathbf{k}}}{2t^2} + \frac{1}{2}\left( t^{-\frac{2}{3}} |\bar{\mathbf{k}}|^2 + m^2 \right) \phi^2_{\mathbf{k}} \right] \right)}. \tag{35}$$

where $\bar{\mathbf{k}} = \mathbf{k} V_0^{\frac{1}{3}}$ is the (scale invariant) wave vector. Upon quantization the corresponding operator has the spectrum

$$E_n = \sqrt{\frac{8}{3}\left[ \Lambda + \sum_{\mathbf{k}} \left( n + \frac{1}{2} \right) \omega_{\mathbf{k}}(t) \right]}, \tag{36}$$

$$\omega_{\mathbf{k}}(t) = \frac{1}{t}\sqrt{t^{-\frac{2}{3}}\bar{\mathbf{k}}^2 + m^2}. \tag{37}$$

To find the vacuum energy density of the matter modes we again set $n = 0$, and consider the massless case $m = 0$ for simplicity. The $\sum_{\mathbf{k}}$ is a sum over comoving modes, which is evaluated by converting the sum to an integral in the usual way with a $k-$space volume $d^3k$:

$$\sum_{\mathbf{k}} \omega_{\mathbf{k}} \to \frac{1}{t^{\frac{4}{3}}} \int_0^{\bar{k}_p} \frac{d^3\bar{k}}{(2\pi)^3}\ \bar{k}. \tag{38}$$

Restoring factors of Planck mass, this gives for vacuum energy density the result

$$\rho_{vac} = \frac{E_0}{t} = \frac{\rho_P}{\bar{t}}\sqrt{\frac{8}{3}\left( \Lambda l_p^2 + \frac{1}{8\pi^2 \bar{t}^{\frac{4}{3}}} \right)}, \tag{39}$$

where $\bar{t} = t/t_P$. The overall factor is the same as that for the homogeneous case. The peculiar time factor multiplying the second term in the square root comes from the mode frequency (37), which in turn has its origin in the scalar field gradient term $\sqrt{q}q^{ab}\partial_a\phi\partial_b\phi \to a^3|\mathbf{k}|^2/a^2$.

It is apparent that the general features of the homogeneous case, the square root and explicit time dependence are still present. We may again compute a numerical estimate for the vacuum energy by substituting on the r.h.s. of (39) the present age of the universe $\bar{t} = 10^{61}$ and the $\Lambda$ value from (7). This gives

$$\rho_{vac} \sim \rho_P \times 10^{-103} = 5 \times 10^{-7} \mathrm{K}g/\mathrm{m}^3. \tag{40}$$

This formula makes clear that the present vacuum energy density with the global choice of volume time is far

smaller than the huge value from standard arguments. It shows that there is no cosmological constant problem.

Let us summarize our main result. We find for the non-perturbative matter-gravity system in the cosmological context that the physical Hamiltonian (i) is not a linear function of $\Lambda$, (ii) is explicitly time dependent, and (iii) yields the explicit formula (39) for vacuum energy density. A numerical evaluation of this density shows that the vacuum energy problem is absent due to the time suppression factor. Beyond these details, our general argument reveals that there is an intimate connection between time, vacuum energy and the cosmological constant, which is revealed by extracting the physical Hamiltonian for a matter-gravity system is a physically reasonable time gauge. (For negative cosmological constant our results do not apply above a critical time value. This means that the volume time gauge does not provide a useful foliation.)

In closing we provide several comments on our approach, pointing out what we think are generic features and what are limitations which merit further work.

(i) In FRW cosmology the matter energy density is identified as the right hand side of the Friedmann equation. This is fine for classical theory, but a non-perturbative quantum theory requires a physical Hamiltonian for the full matter-gravity system before one can talk about the true vacuum.

(ii) Our approach does not address the question of why the observed cosmological constant is so small. But it does address the problem of the relation between vacuum energy density and the cosmological constant; this we show is time dependent and non linear.

(iii) The functional form of the physical Hamiltonian, and hence the vacuum energy density is dependent on the time gauge. The square root and time dependent physical Hamiltonian are a common feature of canonical time gauge fixing. This is because the Hamiltonian constraint is quadratic in momenta for usual matter fields (see [14] for an unusual exception). As a result one ends up solving at least a quadratic equation for the momentum conjugate to the chosen time variable.

(iv) Our results are derived in only one time gauge in the setting of FRW cosmology with perturbations. Although this is observationally relevant, for more general metrics it is not possible to use volume time because it does not provide a complete time gauge fixing. The general problem is more challenging. It requires fixing a suitable local matter or geometry scalar as time, and deriving the corresponding Hamiltonian density. The latter may not be a simple function, and the spectrum problem correspondingly difficult.

(v) Beyond the homogeneous case, our development uses the fixed volume time gauge from the background to define the physical Hamiltonian of matter perturbations. The spectrum of this Hamiltonian provides only the energy part of the semiclassical equation (1). The pressures can be computed, and would come from analyzing the spatial diffeomorphism constraint $D_b\pi^{ab} = j^a(\phi)$ to leading order beyond the homogeneous approximation (where this constraint is trivially satisfied). This would be among the necessary steps for developing a canonical semiclassical approximation using our approach as a starting point.

(vi) We used a Planck scale cutoff in deriving the vacuum energy density (39). Our justification of this is the same as that in the usual treatment because the scalar perturbations are effectively being treated on the FRW background. That is, it is not yet full quantum gravity. But the novel feature in the formula, unlike the flat space case, is the time factor suppression of this term in (39), which leaves the $\Lambda$ factor as the dominant one at late time.

(vii) What becomes of the "low energy" CC problem in a small patch of spacetime where there is a local timelike Killing vector field? This local Minkowski time is obviously fine for short timescale particle physics during which the universe does not expand much. But our approach and results suggest that it is not useful to pose questions such as "does the vacuum gravitate" in a local flat patch of a cosmological spacetime.

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